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Relationship between the yearly maxima of peak and daily discharge for some basins in Tuscany

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ABSTRACT The paper deals with the estimation of the probability distribution of the yearly maximum of the peak discharge Q by means of the distribution of the maximum of the daily discharge q and the distribution of the ratio R = Q/q. The study was carried out for some catchments in Tuscany, analysing the dependence of the parameters of R on the geomorphic catchment parameters. The values of the discharge Q, relevant to an assigned return period, obtained by the methods given herein agree rather well (the error is about 20%) with those directly obtained from the observed values of Q.

Relation entre le maximum annuel des pointes de débit et le débit journalier sur certain bassins versant de Toscane

RESUME Cette communication traite de la détermination de la distribution des probabilités du maximum annuel des pointes de débits Q à partir de la distribution du maximum des débits journaliers q et de la distribution du rapport R = Q/q. L'étude a été effectuée pour certaines bassins versants de Toscane en analysant l'influence des paramètres géomorphologiques du bassin sur les paramètres de R. Les valeurs du débit Q correspondant à une période de retour donnée, obtenues par la méthode qui est décrite ici sont en assez bon accord avec celles obtenues directement à partir des valeurs observés Q (l'erreur est de 20% environs).

INTRODUCTION

The estimate of peak flood discharges is one of the most important problems in hydrology and one that has long been a subject of research. In some cases a satisfactory solution can be easily worked out, whereas in others the problem remains unsolved. For instance, when a considerable series of data related to peak flood discharges in the section in question is available, the problem can nowadays be solved by regarding the yearly maximum discharge as a random variable and identifying a probability distribution that should be convenient for data interpretation. Generally, however, in the case of most water courses maximum daily discharge data are available in greater number than peak flood discharge data. This is due to the fact that, in Italy at least, peak flood discharge values are more rarely published than mean daily discharge values, let alone the total lack of such data in many cases. In this connection it should be noted that the so-called mean daily discharges are actually often derived from very few or even a single reading of the staff gauge at a fixed hour of the day. However, from a statistical point of view, as the error can be either positive or negative in sign, these values can be rationally assimilated to the actual mean daily discharges.

The problem is then to proceed from the estimate of the maximum daily discharge q with an assigned return period T (which can be easily worked out when a fairly long series of observations is available) to the maximum peak discharge Q with an equal return period. This problem can usually be solved through regional research enabling results to be extrapolated to the water course for which peak discharge is to be estimated. We shall here mention the results of some studies which have been carried out on Italian regions of variable size.

In this context we first recall the well-known formula proposed by Tonini (1939) for the whole set of Italian rivers. In this formula, as well as in the following ones, S represents the basin area in km^2:

$$Q/q = 1 + 68\ S^{-0.5} \qquad (1)$$

Secondly, we recall Cotecchia's (1965) formulae, also for Italian water courses:

$$Q/q = 32\ S^{-0.313} \qquad (2)$$

$$Q/q = 16\ S^{-0.19} \qquad (3)$$

They hold, respectively, for areas larger or smaller than 120-140 km^2. They have been derived as curves enveloping the observed maxima and provide a constant value of the ratio Q/q, which is independent from the value assigned to the return period.

The ratio between peak discharge and daily discharge with a given return period T can be easily worked out from Lazzari's (1967, 1968) formulae for basins in Sardinia. For basins located on the west side of the island:

$$Q(T)/q(T) = 1.35\ (1.08^{z(T)}) \qquad (4)$$

For those located on the east side:

$$Q(T)/q(T) = 1.36\ (1.07^{z(T)}) \qquad (5)$$

where z(T) is the value of the standardized Gaussian variable with *return period* T.

A study carried out by Tonini *et al.* (1969) for some basins in the Dolomites shows:

$$Q(T)/q(T) = 2.39\ S^{-0.112} \tag{6}$$

Versace & Principato (1977), who have been recently engaged in examining this problem, have found that for 16 basins in Calabria (all but one with an area smaller than 600 km^2) the ratio Q(T)/q(T) is a random variable, whose distribution is lognormal and independent of S.

In a study on peak flood discharges of basins in Liguria and Tuscany (Canuti & Moisello, 1980) we investigated the dependence of the ratio Q(T)/q(T) on basin geomorphic parameters. If the probability distributions of the maximum peak discharges Q and the maximum daily discharges q are assumed lognormal (or Gumbel distributed) and if the coefficient of variation of Q is assumed equal to that of q, the ratio Q(T)/q(T) does not depend on T and is equal to the ratio between the means:

$$\lambda = E(Q)/E(q) \tag{7}$$

Actually, the assumption that the coefficients of variation of Q and q should be equal (which is approximately the case) implies that the ratio between Q and q is not independent of the values of q. Otherwise, always for equal coefficients of variation, this ratio would be a single value function of q. In the above-mentioned study the following multiplicative formula expresses the dependence of the ratio λ on basin parameters:

$$\lambda = 11.72\ S^{-0.100}\ H_m^{-0.181} \tag{8}$$

where H_m is the mean height of the basin in m.

Since, in practice, one should not necessarily adopt the above hypotheses, it was considered worthwhile to carry out some research in this field and the results are reported in the present paper.

APPROACH TO THE STUDY

Our approach to the study was as follows. Q is the maximum yearly peak discharge, q the mean daily discharge and R the ratio Q/q. Q and q as well as R can be treated as random variables. As will be shown later, this implies that by knowing the parameters characterizing the distributions of q and R (i.e. in this case, the mean and the standard deviation) it is possible to estimate the parameters characterizing the distribution of Q.

In the case where only a series of observations related to the daily discharge q is known, the problem is then that of estimating the parameter values characterizing the distribution of R. Through a regional analysis this study aimed to define how these parameters depend on basin geomorphic characteristics, in order to provide a method of working out the distribution of Q from that of q through the estimates of the parameters of R.

The basic point of this study is therefore the dependence of these parameters on basin geomorphic parameters. With this in mind, our investigation is connected with the geomorphic quantitative analysis of basins and their drainage network, represented by

Horton's (1932, 1945) and, subsequently, Langbein's (1947) and Strahler's (1957) contributions to hydrology. These authors proved that the hydrological characteristics of a river are significantly related to the morphological characteristics of the corresponding basin. Their studies started a new trend in research which has been widely developing in the United States and elsewhere (Ghose & Pandy, 1963; Avena *et al.*, 1967; Gregory & Walling, 1973; Christofoletti, 1969).

Among the different objectives we shall cite: the understanding of cause-effect relationships between morphometric and hydrological (Leopold & Miller, 1956; Melton, 1958; Carlston, 1963) or climatic (Chorley, 1957) characteristics; the evolution of the drainage pattern and the resulting relationship between erosion and sedimentation (Schumm, 1956, 1977a; Leopold & Langbein, 1962; Morisawa, 1964; Shreve, 1966, 1967; Smart, 1972); the morphologically based forecast of runoff and sediment transport characteristics in a basin (Hickok *et al.*, 1959; Hadley & Schumm, 1961; Morisawa, 1962; Gray, 1965; Gregory & Walling, 1973; Schumm, 1977b).

A smaller part of these studies concerns the correlation between peak discharges and basin hydro-geomorphic factors. Major contributions in this field are represented by Beard's work on flood potential (1975) and Patton & Baker's (1976) work on the relationship between morphometry and maximum discharges in small basins (100 mile^2) in areas with different flood potentials. We have already mentioned one of our studies on this subject.

As regards the relationships between flood hydrographs and geomorphic parameters in particular, attention should be paid to the work of Hickok *et al.* (1959) and to the more recent ones of Gregory (1979), Rodríguez-Iturbe & Valdés (1979) and Valdés *et al.* (1979).

In small experimental basins located in dry areas Hickok *et al.* (1959) observed that the hydrograph time to peak and the peak discharge/flood total volume ratio show the best correlation with the land slope, distance from outlet and length of the basin portion ("source area") having the steepest land slope and with the drainage density of the whole basin.

According to Gregory, the basic geomorphic characteristics of a basin related to maximum discharges are the drainage network density and another parameter. For peak discharge the significant parameter proposed by the author is drainage network power, in which basin relief is combined with the product of the channel cross section multiplied by channel length. The author shows that interesting results can be obtained but, as he measures 369 sections in 14 basins, his method has only a limited practical application.

The work carried out by Rodríguez-Iturbe & Valdés (1979) and by Valdés *et al.* (1979) proposed and successfully verified an IUH (instantaneous unit hydrograph) in which the maximum ordinate q_p is a function of the basin length L, of Horton's number R_L (the ratio between the lengths) and of the current velocity ν (which accounts for nonlinearity phenomena), while the time to peak t_p is a function of Horton's numbers R_A (the ratio between the areas) and R_B (bifurcation ratio), besides being a function of L, R_L and ν.

As basin parameters this study has employed eleven common geomorphic parameters. First of all, three basic geometric parameters have been used: length (measured along the drainage line), mean altitude (above sea level) and area of the basin. This study relied on the values calculated and published by the Italian Hydrographic Service.

For the variables expressing the form of the basin, we chose to adopt the elongation ratio (Schumm, 1956) (the ratio between the diameter of a circle having the same area as the basin and the basin length) and the circularity ratio (Miller, 1953) (the ratio between the basin area and that of a circle of equal perimeter).

Three parameters have been used to express the basin relief, whose importance had already been pointed out in Horton's early studies (Horton, 1932, 1945): basin relief (maximum basin altitude at the basin outlet), relief ratio (Schumm, 1956) (the ratio between the mean altitude of the basin perimeter referred to the outlet and the basin maximum length) and the slope of the main stream (the ratio between the basin relief and the length of the channel).

The drainage network organization has been represented by three parameters: basin order (Strahler, 1957), basin magnitude and frequency of first order streams (the ratio between basin magnitude and basin area) (Morisawa, 1962).

A fundamental problem in the studies of quantitative geomorphology is the method of survey and measurement of geomorphic parameters related to drainage network organization and frequency. It is commonly accepted that the survey scale affects the number, order and density of the streams (Leopold & Miller, 1956; Patton & Baker, 1976; Dramis & Gentili, 1977); the method of survey (Del Sette & Fastelli, 1979) is also a determining factor. We have chosen the method that seems the best (Sfalanga *et al.*, 1972): that is photo-interpretation of the drainage network, by air photos 1:13 000 scale, reported on a 1:25 000 topographic map. Although this method is certainly difficult and somewhat subjective, a ground survey cannot really be proposed.

Horton's early works point out the importance of drainage density, i.e. the ratio between the total length of the streams and the drained area. This parameter is in its turn controlled by several variables (Horton, 1945) and its correlation with various parameters has been widely studied (Schumm, 1956; Melton, 1958; Carlston, 1963; Chorley, 1957; Hadley & Schumm, 1961).

This parameter is particularly relevant, at least from a theoretical point of view, but so difficult to estimate (Patton & Baker, 1976) that its real importance has been questioned (Dingman, 1972). For this reason recent studies (Gardiner, 1979; Richards, 1979) have tended to overcome the difficulty of measuring the drainage density by successfully replacing it with other parameters, such as the number of streams or the number of stream junctions. This is also the method used in the present study, in which the areal frequency of first order streams has been taken into account. In fact we have been able to check that there is an actual correlation between this parameter and drainage density, as previously shown by other authors (Dramis & Gentili, 1975), for the basins and sub-basins of streams whose drainage density had already been calculated (Sieve, Pesa, Elsa, Greve, Era) (Sfalanga *et al.*, 1972; Canuti *et al.*, 1975, 1979a, b).

As far as discharge data are concerned, the lack of published data for the basins under consideration has been overcome by using records from stage recorders.

As a rule, the data should have been the yearly maximum peak discharge Q and the yearly maximum daily discharge q, obtained from two different flood events. However, to carry out the work more quickly, the data taken into consideration were Q and q corresponding to the same flood. Errors are actually limited because generally the yearly maxima of both discharges are observed during the same event.

For some basins the stage recorders have worked for only a few years. To increase the amount of data available, so that all the discharge series comprise at least 10 values, in some cases it was thought appropriate to use the records of several floods relevant to the same year, instead of only the yearly maximum floods. This approximation can be justified since it can be reasonably assumed that the distribution of the ratio R between peak discharge and mean daily discharge of the same flood (which is the case under consideration) characterizes a certain range of discharge values, whether they are the maximum yearly values or not.

RELATIONSHIP BETWEEN THE DISTRIBUTION PARAMETERS OF PEAK DISCHARGE AND THE CORRESPONDING DAILY DISCHARGE

If Q is peak discharge, q the corresponding daily discharge and R the ratio between them,

$$Q = Rq \tag{9}$$

These three variables can all be considered random variables.

Generally, the distribution of the variable Q can be determined once the joint distribution of the variables R and q is known. The relationships linking the parameters characterizing the distribution of Q and those characterizing the joint distribution of R and q become particularly simple when the variables R and q are independent. In fact, the mean of the product of two independent variables equals the product of the means of the two variables. If E(Q), E(R) and E(q) are the means of the variables under consideration then

$$E(Q) = E(R)\ E(q) \tag{10}$$

The variance of Q is defined by the expression:

$$\mathrm{var}(Q) = E(Q^2) - E^2(Q) \tag{11}$$

Since the above relation of the mean of the product of the variables R and q is obviously valid also for the product of their squares, there follows:

$$\mathrm{var}(Q) = E(R^2)\ E(q^2) - E^2(R)\ E^2(q) \tag{12}$$

By deriving the expressions of $E(R^2)$ and $E(q^2)$ from the definition relations as follows:

$$\mathrm{var}(q) = E(q^2) - E^2(q) \tag{13}$$

$$\mathrm{var}(R) = E(R^2) - E^2(R) \tag{14}$$

and by substituting them in the expression of var(Q), we obtain:

$$\mathrm{var}(Q) = \mathrm{var}(R)\ \{\mathrm{var}(q) + E^2(q)\} + E^2(R)\ \mathrm{var}(q) \tag{15}$$

It is to be noted that the hypothesis that the variables R and q should be independent is the only requirement for the validity of expressions (10) and (15), regardless of the distribution of R and q, which might even be unknown.

Table 1 lists the 18 basins considered and Fig.1 shows their locations. For all of them, to prove the hypothesis that R and q are independent, we have studied the distribution of a transform of the observed value of the linear correlation coefficient r between the logarithms of the variables R and q. The transform is expressed as follows:

$$t = \frac{r\sqrt{N-2}}{\sqrt{1-r^2}} \tag{16}$$

Table 1

River basins	Drainage basin area (km²)	Strahler basin order	Basin magnitude	First order channel frequency	Basin length (km)	Mean basin altitude (m)	Relief ratio	Basin relief (m)	Circularity ratio	Elongation ratio	Channel slope
Arno at Stia	62	7	3 053	49.2	11	891	0.061	1214	0.760	0.807	0.110
Arno at Subbiano	738	8	28 013	37.9	45	720	0.023	1407	0.530	0.680	0.031
Arno at Pollino	445	8	17 991	40.4	29	801	0.031	1344	0.620	0.820	0.046
Sieve at Bilancino	150	7	17 242	114.9	18	475	0.032	905	0.267	0.760	0.050
Sieve at Fornacina	831	8	58 840	70.0	58	490	0.015	1565	0.420	0.560	0.027
Greve at Falciani	120	7	7 678	63.9	26	386	0.017	795	0.517	0.475	0.030
Pesa at Sambuca	119	7	7 608	63.9	26	454	0.016	707	0.415	0.473	0.027
Bisenzio at Praticello	54	6	4 153	76.9	10	684	0.071	1014	0.553	0.828	0.010
Bisenzio at Gamberame	150	7	10 783	71.9	26	565	0.034	1183	0.580	0.530	0.045
Bisenzio at Carmignanello	100	7	7 503	75.0	16	630	0.048	1112	0.570	0.705	0.069
Elsa at Castelfiorentino	806	8	23 043	28.5	53	243	0.008	677	0.340	0.680	0.013
Nievole at Colonna	32.5	6	4 095	126.0	13	390	0.038	835	0.425	0.490	0.064
Era at Capannoli	335	8	32 402	96.7	37	225	0.010	650	0.335	0.550	0.017
Vincio at Cireglio	1.38	4	81	58.7	3.2	764	0.060	430	0.270	0.662	0.216
Pescia at Molino Narducci	47	7	2 800	59.5	10	658	0.063	931	0.614	0.773	0.093
Scheggia at Ponte Vertelli	39	7	1 696	43.5	8	996	0.083	1011	0.544	0.880	0.126
Brana at Burgianico	13	5	330	25.3	7	445	0.056	1044	0.504	0.581	0.149
Terzolle at Le Masse	14	6	696	49.7	4.8	309	0.060	664	0.780	0.879	0.138

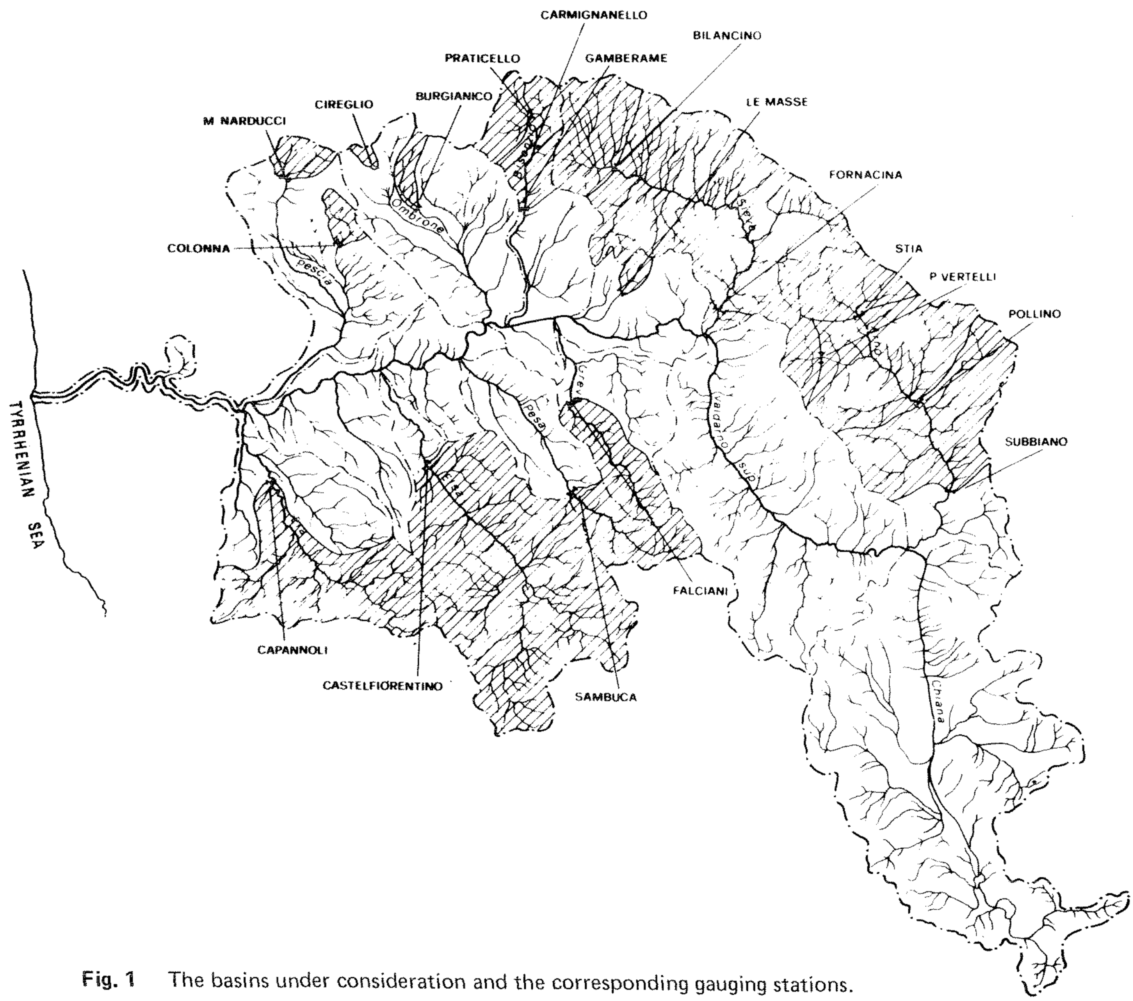

Fig. 1 The basins under consideration and the corresponding gauging stations.

where N is the sample size. If the correlation coefficient, whose estimated value is r, is actually zero and the distributions of the logarithms of R and q are normal, the transform is a Student t with (N - 2) degrees of freedom.

The values of t were computed for the 19 basins listed in Table 2 (in all the analyses not using geomorphic parameters the Rio Sana ad Cartiera Valgiano was also included). These values are much more scattered than was expected: in the probability interval 0.05-0.95 only about 70% of the values are included. The mean of t is -0.058. The fact that this value is negative could support the hypothesis, referred to later, that there is a negative correlation between the variables R and q. As both positive and negative values of t occur almost equally, the independence hypothesis can be accepted, at least for practical purposes.

We shall not overlook that this is a workable hypothesis for values of R and q related to the same stream. In other words, the variability of the ratio R between the peak discharge and the

Table 2

River basins	$\bar{Q}$ ($m^3 s^{-1}$)	s (Q) ($m^3 s^{-1}$)	$\bar{q}$ ($m^3 s^{-1}$)	s (q) ($m^3 s^{-1}$)	$\bar{R}$	s (R)
Arno at Stia	69.0	32.2	35.4	37.1	1.81	0.43
Arno at Subbiano	455.2	221.0	235.2	80.7	1.93	0.57
Arno at Pollino	249.6	99.3	159.9	44.3	1.53	0.38
Sieve at Bilancino	201.6	142.5	76.2	57.3	2.77	0.94
Sieve at Fornacina	500.7	269.3	271.7	200.8	1.96	0.36
Greve at Falciani	77.6	48.0	32.1	14.4	2.28	0.59
Pesa at Sambuca	19.3	7.9	9.4	2.2	2.04	0.67
Bisenzio at Praticello	51.9	19.5	24.6	8.6	2.11	0.50
Bisenzio at Gamberame	103.8	74.0	52.0	36.5	2.03	0.84
Bisenzio at Carmignanello	96.1	34.3	48.9	22.9	2.04	0.44
Elsa at Castelfiorentino	202.8	138.8	99.9	100.3	2.35	0.55
Nievole at Colonna	16.6	6.1	9.6	4.9	1.91	0.64
Era at Capannoli	203.5	79.1	135.3	62.8	1.57	0.34
Vincio at Cireglio	3.4	1.7	1.5	0.6	2.15	0.60
Pescia at Molino Narducci	43.0	28.9	23.0	10.9	1.77	0.56
Scheggia at Ponte Vertelli	17.8	7.2	9.0	2.4	1.95	0.42
Brana at Burgianico	22.9	19.1	8.5	3.7	2.44	1.18
Terzolle at Le Masse	5.2	3.0	2.1	0.85	2.95	1.74
Rio Sana at Cartiera Valgiano	1.29	0.70	0.84	0.36	1.49	0.49

corresponding daily discharge is so great, compared to the variability of the maximum yearly value of the daily discharge, that it is practically impossible to observe greater regularity in the flood hydrographs to which the highest mean daily discharges correspond. This effect can be clearly seen when, if different rivers are considered, one proceeds from a given river to another with a larger basin, for which critical rainfalls are more regular and the reservoir action of the basin is more marked.

The coefficients of variation of Q, R and q (i.e. the ratios between standard deviations and means) are interesting. If R and q are assumed independent, from the definition relation of variation coefficient and from formulae (10) and (15) the following relation can be easily derived:

$$CV^2(Q) = CV^2(q)\ \left[CV^2(R) + 1\right] + CV^2(R) \qquad (17)$$

This relation implies that the variation coefficient of Q is always higher than that of q. Obviously, this is valid for the theoretical distributions of the variables under consideration: the values of the empirical parameters which have been observed in single samples can satisfy this relation only approximately.

Figure 2 shows the experimental points representing the observed values of CV(Q) and CV(q): they are considerably scattered. On the average, the observed value of CV(R), which of course varies from case to case, equals 0.28. Figure 2 shows the curve along which the points should be aligned if the value of CV(R) were constant and equal to 0.28. It can be seen that the experimental values of CV(Q) are on average lower than the curve, though slightly higher than those of CV(q).

Incidentally, we shall note that if R and q are independent the fact that CV(Q) and CV(q) are equal implies that CV(R) equals zero and therefore R is a constant.

In case the independence hypothesis is not valid it is better to proceed to the logarithms. The relation (9) then becomes

$$\ln Q = \ln R + \ln q \tag{18}$$

There follows:

$$\mathrm{var}(\ln Q) = \mathrm{var}(\ln R) + \mathrm{var}(\ln q) + 2\,\mathrm{cov}(\ln R, \ln q) \tag{19}$$

It is evident that the variance of the logarithm of Q (and consequently also of Q) is lower than the variance corresponding to the case in which R and q are independent, if the covariance of the logarithms of R and q (and consequently also of R and q) is negative, as one would reasonably expect.

Actually, the mean variance of ln Q is slightly lower than the sum of the variance of ln R and ln q, as shown in Fig.3. To this aim, these variances have been calculated with the method of moments on the assumption that the distribution of the original variables should be lognormal.

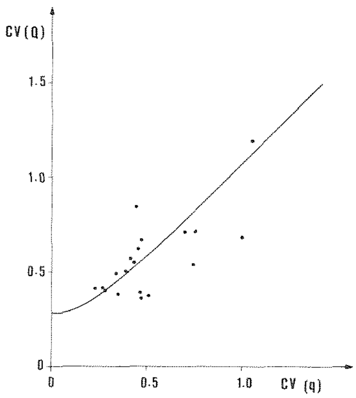

Fig. 2 Relationship between variation coefficients CV(Q) and CV(q). Observed values and theoretical curve for CV(R) constant value.

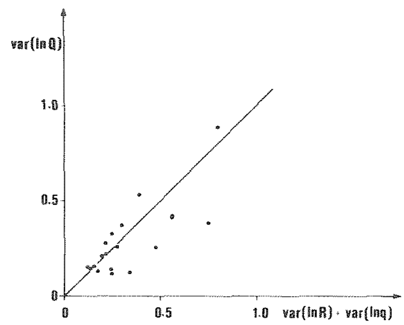

Fig. 3 Comparison between the observed values of var (ln Q) and of the sum [var (ln R) + var (ln q)].

To conclude, we can reasonably hold that there is a negative correlation between the variables R and q. However, since this correlation is very low and the results prove very doubtful, owing to the limited dimensions of the samples from which the parameters have been calculated, for practical purposes it can be accepted that R and q should be schematized as independent variables. Moreover, this procedure also simplifies the calculations.

It was also thought useful to investigate the type of probability law according to which the ratio R is distributed. The results have shown that generally the distribution of R can be correctly interpreted with the two-parameter lognormal law:

$$z = a \ln R + b \tag{20}$$

where z is the standardized Gaussian variable and a and b are the distribution parameters, even though the lower limit of R does not equal zero, as implied in the above expression, but one.

DEPENDENCE OF RATIO R ON GEOMORPHIC PARAMETERS

In the preceding section it was said that the ratio R can approximately be considered distributed according to a lognormal law, whose parameters obviously assume different values which depend on the stream under consideration.

This work examines the dependence of the mean E(R) and the standard deviation $\sigma(R)$ on eleven geomorphic parameters that have been chosen among those most commonly in use. Naturally, E(R) and $\sigma(R)$ values, which are unknown *a priori*, have been substituted with the corresponding estimates $\bar{R}$ and s(R). These values are reported in Table 2, while the values of the geomorphic parameters (for 18 basins only) are reported in Table 1.

As already stated, the geomorphic parameters under consideration are: length, mean altitude and area of the basin; elongation ratio and circularity ratio; basin relief, relief ratio and main stream channel slope; basin order, basin magnitude and frequency of first order streams. Naturally, some of these parameters are reciprocally connected: such are, for example, basin order, basin magnitude and frequency of first order streams; or circularity ratio and elongation ratio.

Not all the geomorphic parameters are equally sampled, as can be inferred from Table 1: for example, while the basin area varies widely and is therefore a good sample, this is not so for the basin order, which varies only slightly from case to case. This must be taken into account when evaluating the results: the fact that a geomorphic variable does not influence E(R) or $\sigma(R)$ is more significant if the variable is well sampled whereas the fact that it influences these parameters is more significant if it is not well sampled.

To study the dependence of the mean E(R) and the standard deviation $\sigma(R)$ on the geomorphic parameters, a combination of backward, forward and stepwise regression procedures has been followed. We shall not go into details but only remark that each procedure allows the selection of the most significant of all the independent variables $x_1, x_2, \ldots, x_p$, for determining the dependent variable y and expressing y as a linear function of the former.

If we assume, without loss of generality, that the first s variables (s not greater than p) are the significant ones the dependent variable derives from the expression:

$$y = a_0 + a_1x_1 + a_2x_2 + \ldots + a_sx_s + \varepsilon \tag{21}$$

where $a_0, a_1, \ldots, a_s$ are the coefficients and ε the multiple regression error.

Each of the above procedures can be adopted as an alternative to the other two and offers different results. With a view to the final choice of the formulae, this study has paid particular attention to what physical significance must be given to the relations that have been found.

Here we shall report and comment on the final results only, totally overlooking the intermediate steps.

A first analysis, which did not exclude any of the 11 geomorphic variables, led to express the mean E(R) as a function of the basin order and the mean altitude, the standard deviation $\sigma(R)$ as a function of the basin magnitude, the mean altitude, the relief, the circularity ratio and the channel slope.

The regression is satisfying in the case of the standard deviation $\sigma(R)$, while it is rather poor in the case of the mean E(R) (the multiple correlation coefficient is equal to 0.876 in one case and to 0.552 in the other).

Since the values of the basin order and magnitude cannot be reasonably calculated to solve practical problems, a second analysis has been carried out, *a priori* excluding from the set of the independent variables the basin order, the basin magnitude and the frequency of first order streams.

In this case the following formulae have been obtained:

$$E(R) = 2.32 - 0.000902\ H_m + 6.10\ R_r \qquad (22)$$

$$\sigma(R) = 1.15 - 0.0127\ L_b - 0.000697\ H_m + 0.000513\ R_b - 0.764\ R_c \qquad (23)$$

where H_m is the mean altitude (in m), R_r the relief ratio, L_b the length (in km), R_b the relief (in m) and R_c the circularity ratio of the basin.

Naturally, the multiple regression coefficients are slightly lower than in the previous case; the slight improvement obtained in the estimate of the parameters of R cannot balance the difficulty of determining such parameters as basin order and basin magnitude. The observed parameter values are compared with the estimated ones in Figs 4 and 5. The following considerations can be made taking into account the signs of the regression coefficients.

First of all, logically the mean E(R) increases, other conditions being equal, if the relief ratio R_r increases, since the reservoir action decreases. Secondly, it is reasonable to link the decrease of $\sigma(R)$, to which an inferior variability of the shape of the hydrographs corresponds, with a greater length of the stream.

It is harder to interpret the role of the other variables. The fact that both E(R) and $\sigma(R)$ decrease if the mean altitude H_m increases might be connected with a more regular rainfall regime at higher altitudes (Canuti & Moisello, 1980). The increase of $\sigma(R)$ with the increase of the basin relief R_b might be connected with a smaller reservoir action and the decrease of $\sigma(R)$ with the increase of the circularity ratio R_c with a more regular (i.e. fan shaped) drainage network.

To check the reliability of peak discharge values, which have been estimated using q and the above results relevant to R, we have compared the observed mean and standard deviation values with those derived from the expressions (10) and (15) in which the mean and variance of R are given the values obtained from formulae (22) and

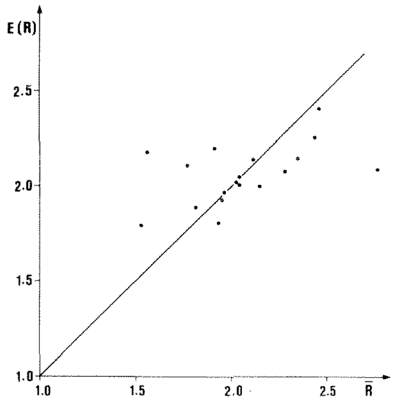

Fig. 4 Regression of R mean on mean altitude and relief ratio: observed value $\bar{R}$, estimated value E(R).

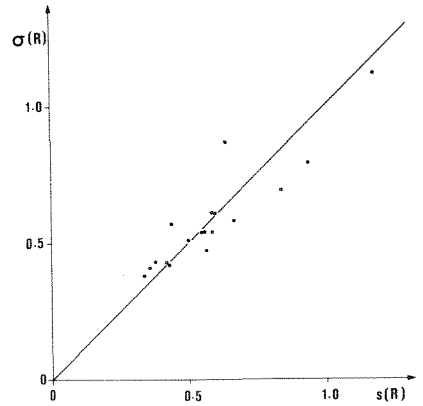

Fig. 5 Regression of R standard deviation on mean altitude, length, basin relief and circularity ratio: observed values s(R), estimated value σ(R).

(23) and the mean and variance of q the observed values. In Fig.6 the estimated values of E(Q) are compared with the observed $\bar{Q}$; in Fig.7 the estimated values of σ(Q) are compared with the observed s(Q).

As regards the mean, there is a marked difference between the estimated value E(R) and the observed value q in the case of the Era at Capannoli (Fig.6); as for standard deviation, there is an even greater difference between the estimated value of σ(Q) and the

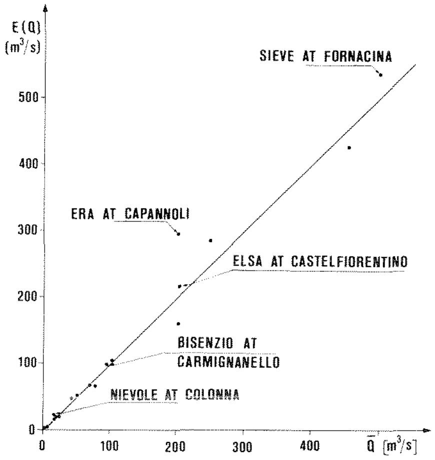

Fig. 6 Mean of peak discharge Q: comparison between observed value $\bar{Q}$ and estimated value E(Q) from mean altitude and relief ratio.

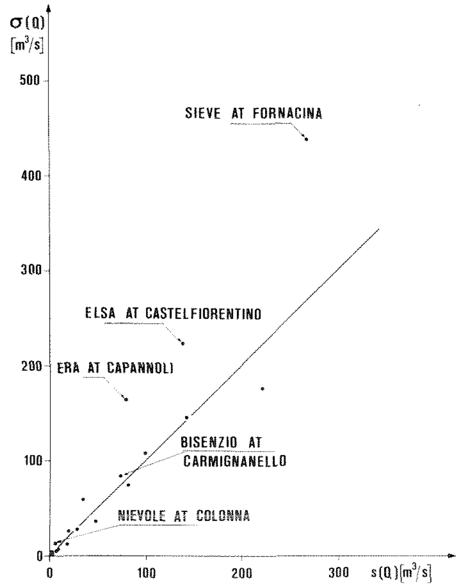

Fig. 7 Standard deviation of peak discharge Q: comparison between observed value s(Q) and estimated value σ(Q) from mean altitude, length, basin relief and circularity ratio.

observed s(Q) in the case of the Era at Capannoli, the Elsa at Castelfiorentino and the Sieve at Fornacina (Fig.7).

These anomalous behaviours can be explained by assuming a non-linear dependence of E(R) and σ(R) on geomorphic variables. In fact, in this case formulae (22) and (23) can be regarded as linearized expressions which are valid only approximately and for a limited range of values of the independent variables. Both E(Q) and σ(Q) increase with E(R) and σ(R) and therefore as the mean altitude H_m decreases. The basins of the Era at Capannoli and the Elsa at Castelfiorentino show very low H_m values in comparison to the other basins. If nonlinearity is assumed the poor results obtained from the above formulae might be due to the fact that the value of the independent variable H_m is at the lower limit of the validity range of the formulae.

Similarly, for the Sieve at Fornacina, we observe that σ(Q) increases with the basin relief R_b, which just in this case rises to a particularly high value. A careful examination of Figs 6 and 7 clearly shows that the values of the mean E(Q) are estimated with a much better approximation than those of the standard deviation σ(Q). This can be fairly easily explained; the estimate of the mean E(Q), given by (10), is noticeably affected by the value of E(q), which of course varies according to the basin size, and is little affected by the value of E(R), which varies only slightly from case to case. For this reason, although the estimate of E(R) on the basis of geomorphic parameters is rather poor (that is, the corresponding regression is poor, as has already been said and shown in Fig.4), this factor does not affect the estimate of E(Q) to any remarkable extent.

Things are different in the case of the standard deviation σ(Q), that can be obtained from the formula (15).

The values of var(R) are more strongly dependent on basin parameters than those of E(R) (the variation coefficient of the observed values is more than double), as can be seen from Table 2. As a result, the sum of var(q) and E^2(q) is multiplied by a term which varies sensibly depending on the basin, whereas var(q), which appears in the second product, is multiplied by a scarcely variable term. The sum in the first product, however, is of the order of four times the variance of q. It is therefore obvious that the errors in the estimate of σ(R) should noticeably affect the estimate of σ(Q).

A further comparison has been carried out in order to make the meaning and limits of this study clearer. First of all, Q values have been assumed distributed according to Gumbel's law. This assumption is made only on the basis that Gumbel's law is very widely used; even in the case that another distribution may fit the data better, it can be reasonably expected that the results of the comparison would not change substantially. Then, the values of discharge Q with a pre-set return period have been estimated in three different ways: first, directly, on the basis of the actually observed values $\bar{Q}$ and s(Q); second, indirectly, on the basis of the values obtained by using the values of E(R) and σ(r) which have been derived from regression formulae (22) and (23) and the observed values $\bar{q}$ and s(q), as reported in Table 2; third, indirectly again, by using the estimates of E(Q) and σ(Q) which are derived from the

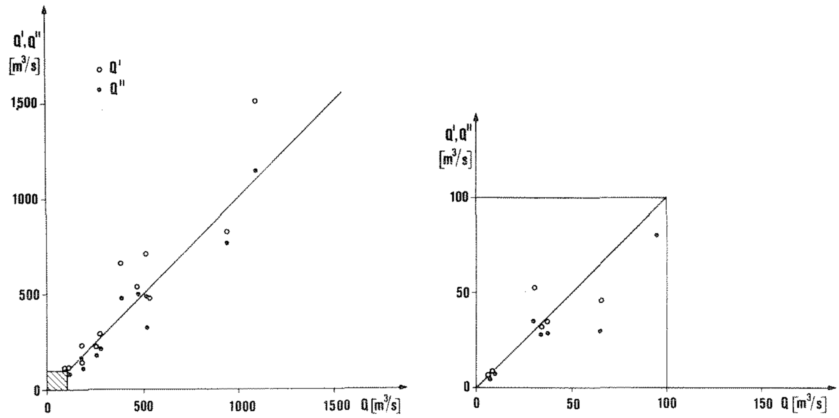

Fig. 8 Comparison between the values of peak discharge Q with a 10 year return period which have been obtained from observed values $\bar{Q}$ and s(Q) and the values Q′ and Q″, obtained from the estimated values E(Q) and σ(Q). Q′ has been derived through mean altitude, length, basin relief, circularity ratio and relief ratio; Q″ has been derived by assigning mean values to E(R) and σ(R).

observed values $\bar{q}$ and s(q) and from the mean values of R and s(R). The latter have been calculated for all the 19 basins that are listed in Table 2. These resulting values are 2.03 and 0.584 respectively.

In Fig.8 the Q values with a 10 year return period corresponding to the observed values $\bar{Q}$ and s(Q) are compared with those obtained from the two indirect estimates of E(Q) and σ(Q). Always in the case of a 10 year return period, Fig.9 compares the absolute values of the relative error of both the indirect estimates, referred to the direct estimates.

An examination of these figures shows that the estimates obtained by using the regression formulae to compute parameters of R are generally slightly better, especially for small basins with an area smaller than 200-300 km^2. A considerable exception to this general behaviour is provided by the Nievole at Colonna and the Era

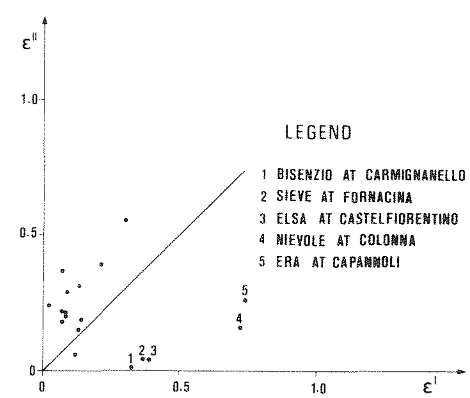

Fig. 9 Comparison between the values of relative errors ϵ' and ϵ'' of indirect estimates Q′ and Q″ of peak discharge with a 10 year return period in relation to direct estimate Q.

at Capannoli, for which the estimates based on the use of the mean values of the parameters of R are much better than those obtained by the regression formulae. Apart from these two special cases, the mean value of the relative error module for a 10 year return period equals 0.16 in the case of regression formulae and 0.22 in the second case. For a return period of 1000 years (often used for design purposes) this mean value equals 0.20 in the former case and 0.23 in the latter.

CONCLUSIONS

Through the data related to a group of basins in Tuscany we have studied the possibility of determining the probability distribution of the yearly maximum peak discharge Q on the basis of the yearly maximum daily discharge. To this aim we have studied the distribution of the ratio:

$$R = Q/q$$

and, moreover, the dependence of the parameters (mean and standard deviation) characterizing the distribution of R on basin fundamental geomorphic parameters.

We have found that the parameters of R can be expressed as a function of basin order, mean altitude, basin magnitude, basin relief, circularity ratio and channel slope.

In the case where basin order and magnitude, which are difficult to determine, should not be taken into account, the parameters of R can be expressed as a function of mean altitude, relief ratio, circularity ratio, length and basin relief. It is possible to obtain satisfactory results also by assigning constant values, equal to the mean values observed for the basins taken into consideration, to the parameters of R.

Basin area does not seem to be one of the most determining geomorphic parameters. However, the distribution of R is affected by the basin size through such parameters as basin magnitude or basin length. The parameters characterizing the distribution of Q can then be worked out as a function of the parameters of q (observed values) and the parameters of R (estimated values).

The discharge values with pre-set return period obtained in this way show a satisfactory agreement with the values directly derived from a statistical analysis of the observed values; the errors are of the order of 20%.

The results that are reported in the present paper can be thought of some interest and may certainly justify a wider study that would concern a great part of the Italian territory.

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